Unsaturated Soil Mechanics for the Study of Rainfall-induced Slope Failures

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ABSTRACT: A significant portion of steep residual soil slopes in the tropics is unsaturated with negative pore-water pressures. The negative pore-water pressures contribute to the shear strength of unsaturated soils. Rainfall-induced slope failures involve infiltration of rainwater and subsequent changes in pore-water pressure and shear strength of soils that are mostly located in the unsaturated zones. Analyses of slope stability problems therefore invoke unsaturated soil mechanics principles. Unsaturated soil mechanics principles and theories relevant to rainfall-induced slope failure studies are discussed. Theories are illustrated through examples of application for the characterization of soil properties, measurement of negative pore-water pressures and seepage and slope stability analyses. Engineered soil covers based on the principles of unsaturated soil mechanics can be used for preventive measures to reduce rainfall-induced slope failures.

1 INTRODUCTION

Residual soils are generally unsaturated and are characterized by high negative pore-water pressures. One of the common problems associated with residual soils is rainfall-induced slope failure. Unsaturated soil mechanics principles are required to fully understand, characterize and analyze behavior of unsaturated soils necessary for various engineering purposes including the rainfall-induced slope failures.

The objective of this paper is to draw attention to the unsaturated soil mechanics principles for the study of rainfall-induced slope failures. Unsaturated soil mechanics principles and theories particularly relevant to rainfall-induced slope failure studies are discussed. Applications of these theories are illustrated through examples.

2 UNSATURATED SOIL MECHANICS

Soils located above the groundwater table are generally unsaturated and possess negative pore-water pressure because of evaporation and transpiration by vegetation. Climatic changes influence the water content of the soil in the proximity of the ground surface. As a result changes occur in the volume and shear strength of the soil. Changes in the negative pore-water pressures associated with heavy rainfalls are the causes of numerous slope failures.

When the degree of saturation of a soil is greater than about 85%, saturated soil mechanics principles can be applied. However, when the degree of saturation is less than 85% it becomes necessary to apply unsaturated soil mechanics principles (Fredlund and Rahardjo, 1987). The transfer of theory from saturated soil mechanics to unsaturated soil mechanics and vice versa is possible through the use of stress state variables.

2.1 Stress State Variables

Stress state variables define the stress condition in a soil and they allow the transfer of theory between saturated and unsaturated soil mechanics. The stress state variables for unsaturated soils (Fredlund & Morgenstern, 1977) are net normal stress \( \sigma - u_a \) and matric suction \( u_a - u_w \), where \( \sigma \) is the total stress, \( u_a \) is the pore-air pressure and \( u_w \) is the pore-water pressure. The stress state in an unsaturated soil can be represented by two independent stress tensors as (Fredlund & Morgenstern, 1977):

\[
\begin{bmatrix}
\sigma_x - u_a & \tau_{xy} & \tau_{xz} \\
\tau_{yx} & \sigma_y - u_a & \tau_{yz} \\
\tau_{zx} & \tau_{zy} & \sigma_z - u_a
\end{bmatrix}
\]

\[
\begin{bmatrix}
(u_a - u_w) & 0 & 0 \\
0 & (u_a - u_w) & 0 \\
0 & 0 & (u_a - u_w)
\end{bmatrix}
\]

where, \( \sigma_x, \sigma_y, \sigma_z \) in Equation 1 are the total normal stresses in the x, y, and z directions, respectively; and \( \tau_{xy}, \tau_{yx}, \tau_{xz}, \tau_{zx}, \tau_{zy}, \tau_{yz} \) are the shear stresses.
Table 1. Summary of classic saturated and unsaturated soil mechanics principles and equations (summarized from Fredlund and Rahardj, 1993b)

<table>
<thead>
<tr>
<th>Principle or equation</th>
<th>Saturated soil</th>
<th>Unsaturated soil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress state variables</td>
<td>$(\sigma - u_w)$</td>
<td>$(\sigma - u_d)$ and $(u_d - u_w)$</td>
</tr>
<tr>
<td>Shear strength</td>
<td>$\tau = c' + (\sigma - u_w) \tan \phi'$</td>
<td>$(\tau = c' + (u_d - u_w) \tan \phi^b + (\sigma - u_d) \tan \phi')$</td>
</tr>
<tr>
<td>Flow law for water (Darcys’ law)</td>
<td>$v_w = -k_w (\partial h_w / \partial y)$</td>
<td>$v_w = -k_w (u_d - u_w) (\partial h_w / \partial y)$</td>
</tr>
<tr>
<td>Unsteady state seepage</td>
<td>$h_w = y + (u_w / \rho_w g)$</td>
<td>$h_w = y + (u_w / \rho_w g)$</td>
</tr>
<tr>
<td>Moment equilibrium</td>
<td>$F_m = \sum \left[ c' \beta R + \left( N - u_w \beta \right) R \tan \phi' \right] / \left( \sum Wx - \sum Nf \right)$</td>
<td>$F_m = \sum \left[ c' \beta R + \left( N - u_w \beta \right) R \tan \phi' \right] / \left( \sum Wx - \sum Nf \right)$</td>
</tr>
<tr>
<td>Force equilibrium</td>
<td>$F_f = \sum \left[ c' \beta \cos \alpha + \left( N - u_w \beta \right) \tan \phi' \cos \alpha \right] / \left( \sum N \sin \alpha \right)$</td>
<td>$F_f = \sum \left[ c' \beta \cos \alpha + \left( N - u_w \beta \right) \tan \phi' \cos \alpha \right] / \left( \sum N \sin \alpha \right)$</td>
</tr>
</tbody>
</table>

### 2.2 Constitutive Equations

Stress state variables are used with measurable soil properties to form single-valued equations known as constitutive equations. Constitutive equations are used to express the relationship between the stress state variables and shear strength or volume change when analyzing soil behavior.

Classical equations for describing the mechanical behavior of unsaturated soils can be presented as an extension of the equations commonly applied to saturated soils. Table 1 summarizes and compares some of the saturated and unsaturated soil mechanics equations pertinent to rainfall-induced slope failure studies. The constitutive equations for unsaturated soils show a smooth transition to the constitutive equations for saturated soils when the degree of saturation approaches 100% or when the matric suction goes to zero. In other words, the saturated soil mechanics is a special case of the general unsaturated soil mechanics.

### 3 STUDY OF RAINFALL-INDUCED SLOPE FAILURES

Application of unsaturated soil mechanics principles to slope stability problems requires assessment of three aspects. These are characterization of soil properties, measurement of negative pore-water pressures, seepage and slope stability analyses.

Six residual soil slopes were selected and instrumented for rainfall-induced slope failure studies in Singapore. The locations of two slopes (Yishun and NTU slopes) are shown in Figure 1. The Yishun slope is located in the granitic Bukit Timah formation while the NTU slope is located in the sedimentary Jurong formation.

Figure 1. Map of Singapore showing geological formations and locations of two instrumented slopes (after Pitts, 1984)

Ground investigations were performed in these slopes by drilling boreholes to establish the stratigraphy of the slope, locate the groundwater table and obtain undisturbed soil samples for laboratory testing. Continuous undisturbed sampling was performed in each borehole using a Mazier sampler.
By combining the information from both field investigations and laboratory tests, simplified soil profiles were produced to provide convenient reference for soil layering and soil properties at the research slopes. The simplified soil profiles for the granitic Bukit Timah (Yishun slope) and sedimentary Jurong (NTU slope) formations are shown in Figures 2 and 3, respectively.

3.1 SWCC & Permeability Functions

An important relationship for an unsaturated soil is the soil-water characteristic curve (SWCC) which relates the water content of a soil to matric suction. The SWCC essentially shows the ability of an unsaturated soil to retain water under various matric suctions. It has a similar role as the consolidation curve of a saturated soil that relates void ratio or water content to effective stress. The SWCC of a soil dictates the manner by which the permeability, shear strength and volume change of the soil will behave at different matric suctions upon drying and wetting (Fredlund and Rahardjo, 1993a).

Unlike the saturated coefficient of permeability $k_s$, the coefficient of permeability $k_w$ in an unsaturated soil is not a constant, it is rather a function of matric suctions. Since water can only flow through the water-filled pores, the SWCC therefore, essentially indicates the space available for the water to flow through the soil at various matric suctions. The unsaturated water coefficient of permeability, $k_w$ can be indirectly estimated from the SWCC and the saturated coefficient of permeability, $k_s$. The shape of the SWCC dictates the variation in the water coefficient of permeability with respect to matric suction or the permeability function. The relationship between soil-water characteristic curve and coefficient of permeability for a sand and a clayey silt is illustrated in Figure 4. The indirect method of obtaining the permeability function is described in detail in Fredlund and Rahardjo (1993b). As SWCC can be determined with greater reliability and in a shorter time, this indirect method of obtaining the unsaturated permeability function is attractive.

![Figure 2. Generalized soil profile of granitic Bukit Timah formation at the Yishun slope (from Rahardjo et al. 2000)](image)

![Figure 3. Generalized soil profile of sedimentary Jurong formation at the NTU slope (from Lim et al., 1996)](image)

![Figure 4. Relationship between soil-water characteristic curve and coefficient of permeability for a sand and a clayey silt (from Fredlund & Rahardjo, 1993a)](image)

A typical soil-water characteristic curve for residual soils from the granitic Bukit Timah formation is shown in Figure 5. A permeability function estimated using the SWCC and $k_s$ is shown in Figure 6 (Rahardjo et al., 2000).
3.2 Seepage Analysis

Seepage analyses are performed to calculate the pore-water pressure changes in a slope due to rainfall. The governing equation (Equation 10 in Table 1) for the flow of water through an isotropic soil can be formulated using Darcy’s law (Equation 8 in Table 1).

A soil slope can be considered as an unsaturated-saturated soil system. The changing pore-water pressures in the slope under the influence of infiltration can be analyzed using Equation 10. The computed pore-water pressures can then be input to a slope stability analysis (Equations 12 and 14 in Table 1), in order to assess the variation in the factor of safety of the slope under different rainfall conditions.

3.3 Shear strength

Consolidated drained triaxial tests provide a direct determination of the effective strength parameters such as the effective cohesion $c'$, effective angle of internal friction $\phi'$ and the $\phi^b$ angle for an unsaturated soil. Typical deviator stress and water volume change versus axial strain curves for residual soils from the sedimentary Jurong formation as obtained from multistage triaxial tests at a constant net confining pressure but varying matric suctions are shown in Figures 7a and 7b (Rahardjo et al., 1994), respectively.

The failure surface is plotted using $(\sigma-u_a)$ and $(u_a-u_w)$ as abscissas. The intersection line between the failure surface and the $\tau$ versus $(\sigma-u_a)$ plane represents the Mohr-Coulomb failure envelope for the saturated condition. On this plane the pore-water pressure is equal to the pore-air pressure or the matric suction is equal to zero. The failure envelope for a saturated soil has a slope and an intercept of $\phi'$ and $c'$, respectively. The total cohesion intercept $c$, at each matric suction was determined from the point where the failure envelope intersects the shear stress versus matric suction plane. The shear strength of a soil increases as the soil becomes unsaturated. The increase in the shear strength can be considered as an increase in the cohesion intercept because of an increase in matric suction (Equation 6b).
The increase in the cohesion intercept with respect to matric suction is defined by the intersection between the failure surface and the $\tau$ versus $(u_a - u_w)$ plane. This line has a slope of $\phi^b$ that can be measured experimentally. The value of $\phi^b$ is generally equal to or less than $\phi'$. The value of $\phi^b$ for residual soils from the sedimentary Jurong formation as obtained from the triaxial tests is shown in Figure 9 (Lim et al., 1996). Figure 9 presents a plot of zone, which encompasses the cohesion intercepts at various matric suctions. The zone of intercept indicates an increase in strength as the matric suction increases. The $c'$ value ranges between 12 to 45 kPa with an average of 28 kPa. Therefore, the mean value of $c'$, interpreted from both the shear stress versus net normal stress plane and the shear stress versus matric suction plane can be taken as 30 kPa (i.e. average of 28 and 32 kPa).

The best-fit line drawn through the centre of the zone indicates that $\phi^b$ is $26^\circ$, which is the same as the $\phi'$, within the matric suction range from 0 to 400 kPa. For matric suctions greater than 400 kPa, the $\phi^b$ angle seems to decrease with an increase in matric suction. This decrease in $\phi^b$ angle from $26^\circ$ at low matric suctions to a value lower than $\phi'$ at matric suctions greater than 400 kPa can be explained from the soil-water characteristic curves and the corresponding degree of saturation of the soil at various matric suctions as shown in Figure 10 (Lim et al., 1996).

Figure 8. Interpretation of $\phi^b$ angle with respect to matric suction from the results of multistage triaxial tests at a constant net confining pressure and varying matric suctions

Figure 9. Cohesion intercept at $\tau$ versus $(u_a - u_w)$ plane (where $(\sigma-u_a)=0$) for fine grained residual soils from Jurong sedimentary formation

Figure 10. Soil-water characteristic curves of residual soils from the sedimentary Jurong formation. ($S$, degree of saturation; $e$, void ratio)

The variation in the slope of a soil-water characteristic curve from initial matric suction to final matric suction depends on the soil type. The sudden increase in the slope of the curve indicates the commencement of desaturation which is referred to as the air-entry value of the soil. Figure 10 illustrates that fine-grained soils (silty clay and organic silty clay) have a higher air-entry value than the coarse grained soils (silty sand). The air-entry value of fine-grained soils falls within 400 to 500 kPa. This implies that the silty clay residual soil remains saturated even at matric suctions of 400 kPa. On the other hand, the air-entry value of silty sand falls within 150 to 250 kPa. For sandy silt specimens the air-entry value falls within 200 to 300 kPa.

A relationship between soil-water characteristic curve and shear strength for a sand and a clayey silt is shown in Figure 11 (Rahardjo et al., 1993a). At low matric suctions, where the suction is lower than the air-entry value of the soil, the soil is at or near saturation condition and the air phase consists of a few occluded bubbles (Corey, 1957). The soil would be expected to behave as though it was saturated. In other words the negative pore-water pressure acts throughout the predominantly water filled pores as in the saturated soil condition. Consequently an increase in matric suction produces the same increase in shear strength as does an increase in net normal stress. As a result, the same values are obtained for $\phi'$ and $\phi^b$.

At matric suctions higher than the air-entry value of the soil, the soil starts to desaturate. The negative pore-water pressure does not act throughout the entire pores as in the saturated soil condition. Therefore, the contribution of matric suction towards the strength of the soil is less than the contribution of the net normal stress at the same stress level. In other words the increase in shear strength with respect to matric suction is less than the increase with respect to net normal stress. Consequently,
the $\phi_b$ value becomes less than $\phi'$ at high matric suctions as observed in Figure 9.

### 3.4 Suction variation with respect to rainfall

As mentioned earlier residual soils are generally unsaturated and the pore-water pressures in the soil are negative. Matric suction is known to play a significant role in slope stability. Therefore, measurement of soil suction is central to the application of unsaturated soil mechanics to the studies of rainfall-induced slope failures. This section briefly outlines theoretical concepts for suction measurement, devices for measuring soil suctions and field instrumentation for rainfall-induced slope failure studies.

#### 3.4.1 Theory

The total suction $\psi$ of a soil is related to the matric suction ($u_d - u_w$) and osmotic suction $\pi$ as (Fredlund & Rahardjo, 1993b);

$$\psi = (u_d - u_w) + \pi$$

(15)

Devices commonly used for measuring soil suction and their range of measurement and principles are listed in Table 2.

#### 3.4.2 Instrumented slope

Instrumentation is an important aspect of stability assessment against rainfall-induced slope failures. The mechanism of water flow within a slope during evaporation and infiltration can be observed through continuous real-time monitoring of the pore-water pressure and rainfall events. The instruments involved are a rain gauge, piezometers and tensiometers for measuring positive and negative pore-water pressures, respectively. Improper installation of instruments into a slope may interfere with the process of infiltration and lead to non-representative

<table>
<thead>
<tr>
<th>Name of device</th>
<th>Suction component measured</th>
<th>Range (kPa)</th>
<th>Principle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensiometers</td>
<td>Pore-water pressures or matric suctions when pore-air pressure is atmospheric</td>
<td>0 ~ 90</td>
<td>Measures negative pore-water pressure directly through a porous ceramic high air-entry cup.</td>
</tr>
<tr>
<td>Thermal conductivity sensors</td>
<td>Matric</td>
<td>0 ~ 400+</td>
<td>Temperature sensing elements in a porous ceramic block measures the rise in temperature of the block, which is inversely proportional to the water content in the porous block. The water content of the porous block can be correlated with matric suction (indirect measurements).</td>
</tr>
<tr>
<td>Psychrometer</td>
<td>Total</td>
<td>100* ~ 8000</td>
<td>Measures the relative humidity of soil. The relative humidity is inversely proportional to the total suction.</td>
</tr>
<tr>
<td>Filter paper</td>
<td>Total / Matric</td>
<td>(Entire range)</td>
<td>A filter paper is calibrated by establishing relationships between its water content at various suctions. When the water vapor in the filter paper is in equilibrium with the water vapor in a soil, the water content of the filter paper is used to compute the total suction of the soil (indirect measurements). The matric suction of the soil is measured when the filter paper is in contact and in equilibrium with the pore-water in the soil.</td>
</tr>
<tr>
<td>Pore fluid squeezer</td>
<td>Osmotic</td>
<td>(No limit)</td>
<td>The pore fluid in the soil at a specific water content can be extracted using pore-fluid squeezer. The osmotic suction of a soil can be determined from the electrical conductivity of the pore fluid of a soil.</td>
</tr>
</tbody>
</table>

* Controlled temperature environment to ± 0.001°C
data. The layout of the instruments is therefore crucial to minimize this interference and to obtain an efficient field data-sampling network.

Figure 12 shows a typical instrument layout for the Yishun slope for rainfall-induced slope failure studies. The slope was instrumented with tensiometers, piezometers, temperature sensors and a rain gauge. All sensors within each slope were monitored automatically by a data acquisition system (DAS), specially designed to meet the need for this study and to withstand the harsh field environment. Data were downloaded from the DAS every four-week.

3.4.3 Monitoring results

The mechanism of infiltration into the residual soil slopes and the effect of climatic conditions (rainfall and evaporation) on the pore-water pressure distributions were of particular interest to this study.

The time series records of pore-water pressure at the study slopes show that significant negative pore-water pressures can develop during prolonged dry periods. A typical short duration time series of rainfall and pore-water pressures at various depths near the crest (row B) of the Yishun slope is shown in Figure 13. The impact of rainfall on the slope’s soil-water condition is reflected in the pore-water pressure changes shown in Figure 13. Negative pore-water pressure developments during dry period and successive development of positive pore-water pressure at various depths in response to rainfall are illustrated clearly in Figure 13.

From the time series of pore-water pressures at all measurement locations, contour plots of pore-water pressure distribution and total hydraulic head distributions across the entire slope profile for key periods were prepared for each slope. Key periods were taken to be: (i) the end of a prolonged dry period when pore-water pressures are at a minimum (i.e., slope stability is at a maximum), (ii) following one or a series of significant rainfall events when pore-water pressures are at a maximum (i.e., slope stability is at a minimum), and (iii) during a significant rainfall event when the pore-water pressure distribution was in transition. These contour plots provided useful information on the magnitude and distribution of pore-water pressures in the slopes during the key periods. The contour plots of pore-water pressure and total hydraulic head distribution for a typical slope (Yishun) during two key periods (dry and wet) are shown in Figures 14 and 15.

Figure 13. Time series of pore-water pressures at various depths at slope crest (row B) in Yishun slope

Figure 14a shows the existence of matric suctions prevailing across the entire slope profile during a dry period (before the occurrence of rainfall events as shown in Figure 13). The slope crest however experienced more suction development than the toe. The contour plot of total hydraulic head distribution for this period as shown in Figure 14b indicates the existence of a hydraulic head gradient between the slope crest and toe, even under a dry condition. As water flow is caused by a hydraulic head gradient, this leads to a downslope migration of water. It is hypothesized that the upslope areas, in addition to draining vertically downwards also contribute to the downslope areas by lateral flows resulting in more drainage upslope and more water retention downslope and thus the lower slope areas remains wetter than the upslope areas. During dry periods in addition to vertical and lateral downslope drainage, combined influence of soil properties, climate, and vegetation encourage high matric suction development in the upslope areas than the adjacent downslope areas. This leads to the variability of pore-water pressure distribution between slope crest and toe as observed in Figure 14a.
Figure 15a shows widespread development of positive pore-water pressure across the slope profile after a series of rainfall events (rainfall events shown in Figure 13). Negative pore-water pressure of lower magnitude may still exist near the slope crest. A comparison of the total hydraulic head gradient prevailing between the slope crest and toe during a dry and a wet period [Figures 14b and 15b] shows that, the hydraulic head gradient during a wet period is more than during a dry period. This means that there is more drainage downslope during a wet period than a dry period.

A limit equilibrium slope stability program, SLOPE/W (Geo-Slope, 1998), was used to determine the critical slip surface and the factor of safety of the slopes from the pore-water pressure distributions monitored in the field during key periods. An example of the variation in the factor of safety with time in response to rainfall for selected key periods is shown in Figure 16 for the Yishun slope.

Figure 16. Variation in factor of safety with time and rainfall for the Yishun slope

4 STUDIES FOR SLOPE STABILIZATION

Unsaturated soil mechanics can also be applied to developing the preventive measures and construction procedures to reduce the possibility of rainfall-induced failures. A method for stabilizing residual soil slopes that shows promise is the development of engineered soil cover (capillary barrier) using the principles of unsaturated soil mechanics. Capillary barrier has been studied and widely used in geo-environmental engineering as a soil cover for landfill (Stormont, 1996). A capillary barrier is a cover system consisting of a fine-grained soil layer placed on top of a coarse-grained soil layer for reducing infiltration into the protected layer. The capillary barrier works on the principle of the distinct difference in the soil-water characteristic curves and permeability functions of the fine and coarse layers under unsaturated conditions. The unsaturated permeability of the coarse layer can be several orders lower than the unsaturated permeability of the fine layer and as a result, this capillary barrier system can prevent water infiltration into the underlying layers.
Rainwater will first enter the fine-grained layer that acts as a storage layer. The high permeability of the fine-grained layer will divert the infiltrating water laterally within this fine layer towards the toe of the slope. Downward movement of water into the coarse-grained layer is more difficult than the lateral diversion along the fine-grained layer due to the much lower permeability of the coarse-grained layer than that of the fine-grained layer. In addition, water is also removed from the fine-grained layer by evaporation and transpiration. The barrier effect works as long as the coarse-grained layer is under the unsaturated condition that produces the low permeability of the coarse layer and the fine layer has sufficient storage for retaining and quick removal of rainwater that enters the system.

A detailed laboratory study on the effectiveness of capillary barrier in reducing infiltration and stabilizing slopes is underway in the Nanyang Technological University. The study includes laboratory experiments in a specially designed infiltration box (Figure 17). The infiltration box is equipped with devices at key locations that can control flux boundary conditions and measure changes in pore-water pressure and water content in the capillary barrier system (see Figure 17). Water balance calculations can also be carried out to examine the effectiveness of the capillary barrier. Such studies are essential in arriving at an optimum design criterion for a capillary barrier.

Typical preliminary investigation results from the study are shown in Figure 18. Variation of pressure head and volumetric water content in three different cross-sections along the fine-grained and the coarse-grained layers of the capillary barrier in association with the changes in flux conditions (precipitation runoff, lateral diversion, and breakthrough) are shown in Figure 18.

CONCLUSIONS

Technology and theory for performing slope stability assessment involving unsaturated soils are available. Reasonable solutions can be obtained when realistic assumptions with respect to matric suctions in the unsaturated zone are used in the analyses. Flux boundary conditions are an important factor when analyzing unsaturated soils that are in direct exposure to the climatic changes. Problems associated with shear strength and volume change can be analyzed and resolved with a consistent unsaturated soil theoretical context thus eliminating the need for many empirical assumptions.

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REFERENCES


Figure 18. Time histories of flux \( q \), pressure head \( h \) and volumetric water content \( \theta_w \) (from Tami et al., 2002)


GLOSSARY

\[ k_w (u_a - u_w) \] = unsaturated coefficient of permeability which is a function of \( u_a - u_w \)

\[ \partial (\sigma_y - u_w) \] = change in effective vertical stress

\[ v_w \] = flow rate of water

\[ k_s \] = water saturated coefficient of permeability

\[ k_w \] = water coefficient of permeability

\[ \partial h_w / \partial y \] = hydraulic head gradient in the \( y \)-direction

\[ \tau \] = shear stress

\[ c' \] = effective cohesion intercept

\[ \phi' \] = effective angle of internal friction

\[ \phi_b \] = angle of shear strength change with a change in matric suction

\[ c \] = cohesion intercept

\[ \partial k_w / \partial y \] = change in water coefficient of permeability in the \( y \)-direction

\[ \partial k_w / \partial x \] = change in water coefficient of permeability in the \( x \)-direction

\[ \partial h_w / \partial x \] = hydraulic head gradient in the \( x \)-direction

\[ F_m \] = factor of safety with respect to moment equilibrium

\[ F_f \] = factor of safety with respect to force equilibrium

\[ R \] = radius of a circular slip surface or the moment arm associated with the mobilized shear force on the base of each slice

\[ W \] = total weight of a slice

\[ N \] = total normal force on the base of the slice

\[ \alpha \] = angle between the tangent to the centre of the base of each slice and the horizontal

\[ \beta \] = sloping distance across the base of a slice

\[ x \] = horizontal distance from the centreline of each slice to the centre of rotation or to the centre of the moments

\[ y \] = elevation head

\[ f \] = perpendicular offset of the normal force from the centre of rotation or from the centre of moments

\[ m_v \] = coefficient of volume change

\[ m_w \] = coefficient of water volume change with respect to change in matric suction

\[ \partial t \] = change in time

\[ \rho_w \] = density of water

\[ g \] = acceleration due to gravity

\[ h_w \] = hydraulic head

\[ u_a \] = pore-air pressure

\[ u_w \] = pore-water pressure

\[ \sigma \] = total stress